Syllabus for a Ph.D. course on
"Regular variation: Basics and beyond"

Anja Janssen

Paris, March 2022

Motivation

Modern extreme value theory is intimately connected to the concept of regular variation: Univariate regularly varying functions describe the domain of attraction of the Fréchet and Weibull max-domains of attraction of the Fisher-Tippett Theorem, multivariate regular variation describes the extremal dependence that we see in random vectors and regularly varying time series provide a convenient framework for the analysis of extremal dependence over time. This course provides an introduction into the basics of regular variation and more recent developments, the latter in particular with a view towards models for (financial) time series.

   Prerequisites: Lecture 1 (2 x 1.5 hours) provides a brief introduction into the basics of regular variation and multivariate extreme value theory. Prerequisite for this lecture is a course on measure theoretic probability theory. Lecture 1 may be skipped by attendants familiar with the mathematical foundations of multivariate extreme value theory.

Lecture 1 (2 x 1.5 hours) Basics of univariate and multivariate regular variation: Regularly varying distribution functions, Convergence of maxima and exceedances, multivariate regular variation, spectral measure.

Lecture 2 (1.5 hours) Regularly varying time series, Part I: The tail process, the time change formula and Markov tail chains

Lecture 3 (1.5 hours) Regularly varying time series, Part II: Extremal index, regular variation of GARCH processes and Breiman's lemma

Lecture 4 (1.5 hours) Refined techniques for asymptotic independence: Asymptotic independence, hidden regular variation, extremes of stochastic volatility processes.