Stochastic Modeling: Exercise Class 4.

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Exercise 1. Consider $I = \int_0^2 x^2 dx$ and 2 different proposals with densities $f_X(x) = 0.5\mathbb{1}_{[0,2]}(x)$ and $f_Y(y) = 0.5y\mathbb{1}_{[0,2]}(y)$.

- Calculate I.
- Compute two MC approximations of I of the form $n^{-1}\sum_{i=1}^n g_X(X_i)$ and $n^{-1}\sum_{i=1}^n g_Y(Y_i)$ for $X_1, \dots, X_n \sim f_X$ and $Y_1, \dots, Y_n \sim f_Y$.
- Compute the corresponding variances $\mathrm{Var}\left(g_X(X)\right)$ and $\mathrm{Var}\left(g_Y(Y)\right)$ and compare the accuracies of the two approximations.

Exercise 2. Consider $I = \int_0^1 \frac{1}{1+x} dx$

- Calculate I.
- Give a MC approximation using the proposal $U \sim \mathcal{U}nif(0,1)$.
- Propose an antithetic approximation.
- Discuss the gain in variance.
- Compute the optimal control variate $c^*(1+U)$.
- Discuss the gain of variance and compare it with the previous one.
- Why cannot we combine the two accelerations?

Exercise 3. Consider $I = \int_0^{10} \exp(-2|x-5|) dx$.

- Calculate I.
- Provide the crude MC approximation.
- Compute its variance.
- Consider an IS approximation with proposal $X \sim \mathcal{N}(5,1)$.
- Compute its weights.
- Discuss the variance gain.