Time Series Analysis: TD3.

Olivier Wintenberger: olivier.wintenberger@upmc.fr

Exercise 1. Show that a MA(q) time series (X_t) is stationary as soon as (Z_t) is.

Exercise 2. Consider a WN(σ^2) (Z_t) and the MA(1) (X_t) defined as

$$X_t = Z_t + \theta Z_{t-1}\,, \qquad t \in \mathbb{Z}\,.$$

Assume that $|\theta| > 1$ and consider the process

$$W_t = \sum_{j=0}^{\infty} (-\theta)^{-j} X_{t-j}\,, \qquad t \in \mathbb{Z}\,.$$

1. Compute $\gamma_X(h)$, $h \geq 0$, the autocovariance function of (X_t).
2. Show that W_t exists in $\mathbb{L}^2$.
3. Express $\mathrm{Var(W_0)}$ in terms of θ and σ^2.
4. Show that (W_t) is WN.
5. Check that we have the relation

$$X_t = W_t + \frac{1}{\theta} W_{t-1}\,, \qquad t \in \mathbb{Z}\,.$$

Exercise 3. This exercise is using the properties of the projection in order to get an efficient algorithm for determining the best linear prediction $\Pi_t(X_{t+1})$ and the associated risk R_t^L. Consider a WN(σ^2) (Z_t) and the MA(1) (X_t) defined as

$$X_t = Z_t + \theta Z_{t-1}\,, \qquad t \in \mathbb{Z}\,,$$

with $|\theta| < 1$.

1. Express the coefficients (φ_j) of the causal solution $X_t = \sum_{j=1}^{\infty} \varphi_j X_{t-j} + Z_t$ of the MA(1) model in term of θ.
2. Deduce $\Pi_\infty(X_{n+1})$ and the associated risk R_∞^L.
3. Show that $\Pi_n(X_{n+2}) = 0$ and $\mathbb{E}[X_{n+1}\Pi_{n-1}(X_n)] = 0$.

4. Deduce from the projection decomposition the recursive formula called

$$\Pi_n(X_{n+1}) = \frac{\sigma^2\theta}{R_n^L}(X_n - \Pi_{n-1}(X_n)), \qquad n \geq 1.$$

5. Deduce the recursive formula $R_{n+1}^L = \sigma^2(1+\theta^2) - \sigma^4\theta^2/R_n^L$ for $n \geq 1$ and the innovation algorithm that update $(\Pi_n(X_{n+1}), R_n^L)$ recursively.